

Research motivation

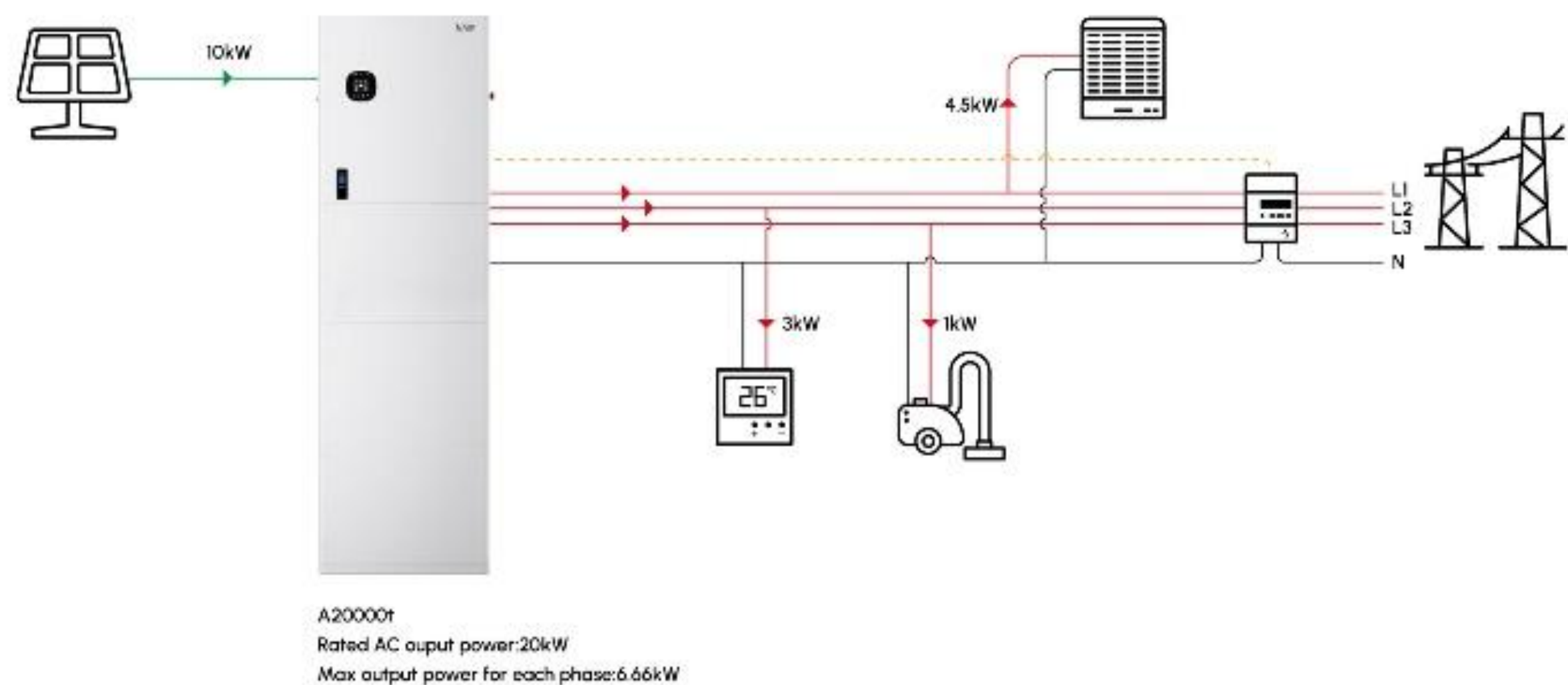
- High renewable penetration brings:
 - Uncertainty in renewable generation
 - Safety limits constraints violation (active/reactive power injection, voltage magnitude)
- Dispatchable region: the maximum capacity of power systems to accommodate renewable generation
 - Solvability (power flow equations)
 - Safety limits (inequalities)

Research objective

Guidance for day-ahead planning for operators. Conventional curtailment is not immediate after security violation.

Research challenge

Constructing the dispatchable region for **unbalanced three-phase** distribution networks remains difficult. Most existing work relies on DC, linearized AC, or single-phase AC models.



Power flow solvability

In [1], [2], backward/forward sweep (BFS) is studied for solving the power flow equations, with convergence guarantees. However, it does not handle safety limits (inequality constraints).

Problem formulation

In [3], the dispatchable region is the feasible range of variable u under the bus injection model (BIM). By lifting $W = VV^H$ and relaxing the rank-one constraint, the formulation becomes a semidefinite program (SDP).

$$\begin{aligned}
 \text{FP}(\mathbf{u}): \min & \sum_{j,\phi} (z_{1j}^\phi + z_{2j}^\phi + z_{3j}^\phi + z_{4j}^\phi + z_{5j}^\phi + z_{6j}^\phi) & (4.4a) & \quad \text{Slack variables} \\
 \text{over } & \mathbf{W} \succeq 0, \quad z_{kj}^\phi \geq 0 \quad (k=1, \dots, 6) & (4.4b) & \quad \text{Dual variables} \\
 \text{s.t. } & \forall j \in \mathcal{N}^+, \phi \in \{a, b, c\} & & \quad \text{Renewable generation} \\
 & \text{tr}(\mathcal{P}_j^\phi \mathbf{W}) - \bar{p}_j^\phi \leq z_{1j}^\phi & (4.4c) & \quad \text{Upper and lower bounds on active power injections} \\
 & -\text{tr}(\mathcal{P}_j^\phi \mathbf{W}) + \bar{p}_j^\phi \leq z_{2j}^\phi & (4.4d) & \\
 & \text{tr}(\mathcal{Q}_j^\phi \mathbf{W}) - \bar{q}_j^\phi \leq z_{3j}^\phi & (4.4e) & \quad \text{Upper and lower bounds on reactive power injections} \\
 & -\text{tr}(\mathcal{Q}_j^\phi \mathbf{W}) + \bar{q}_j^\phi \leq z_{4j}^\phi & (4.4f) & \\
 & \text{tr}(\mathbf{E}_j^\phi \mathbf{W}) - \bar{v}_j^\phi \leq z_{5j}^\phi & (4.4g) & \quad \text{Upper and lower bounds on voltage magnitudes} \\
 & -\text{tr}(\mathbf{E}_j^\phi \mathbf{W}) + \bar{v}_j^\phi \leq z_{6j}^\phi & (4.4h) & \\
 & [\mathbf{W}]_{00} = v_{\text{ref}} & (4.4i) & \\
 & \text{rank}(\mathbf{W}) = 1 & (4.4j) & \quad \text{Definition of dispatchable region:} \\
 & & & \quad U = \{u \in \mathbb{R}_{\geq 0}^6 \mid \text{fp}(u) = 0\}.
 \end{aligned}$$

Non-convex, relax

Dual SDP: u appears only in the objective, under relaxation, this leads to cutting planes that provide an outer approximation.

$$\begin{aligned}
 \text{DP}'(\mathbf{u}): \max & \sum_{j,\phi} \left((\lambda_{2j}^\phi - \lambda_{1j}^\phi) u_j^\phi - \lambda_{1j}^\phi \bar{p}_j^\phi + \lambda_{2j}^\phi \bar{p}_j^\phi - \lambda_{3j}^\phi \bar{q}_j^\phi \right. \\
 & \left. + \lambda_{4j}^\phi \bar{q}_j^\phi - \lambda_{5j}^\phi \bar{v}_j^\phi + \lambda_{6j}^\phi \bar{v}_j^\phi \right) - \text{tr}(\alpha v_{\text{ref}}) \\
 \text{over } & \lambda_{kj}^\phi \quad (k=1, \dots, 6), \quad \alpha \\
 \text{s.t. } & 0 \leq \lambda_{kj}^\phi \leq 1, \quad \mathbf{A}(\lambda, \alpha) \succeq 0 & (8)
 \end{aligned}$$

Exactness of the SDP relaxation



Exactness: the relaxed OPF and original OPF share the same optimal solution and objective value.

Theorem (Sufficient conditions for exactness):

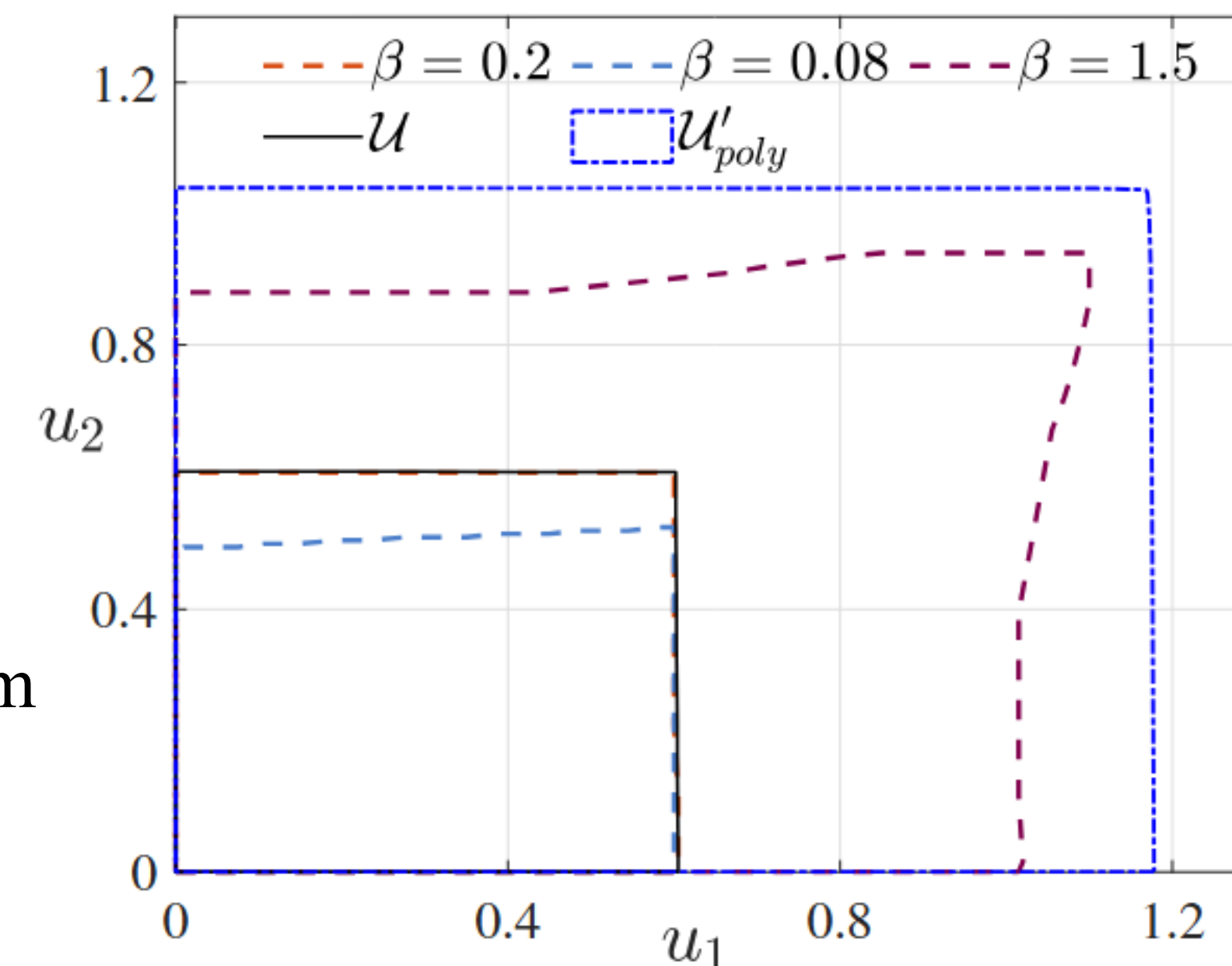
Adding the power loss as a penalty term guarantees a rank-1 solution under the tree network, implying exactness of the relaxation and recoverability of the nodal voltage.

Using the dual PSD matrix B , the KKT condition $\text{tr}(BW)=0$, and the G -invertible matrix property, we prove $\text{rank}(W)=1$, yielding an inner approximation of the dispatchable region.

Investigate the structure of B

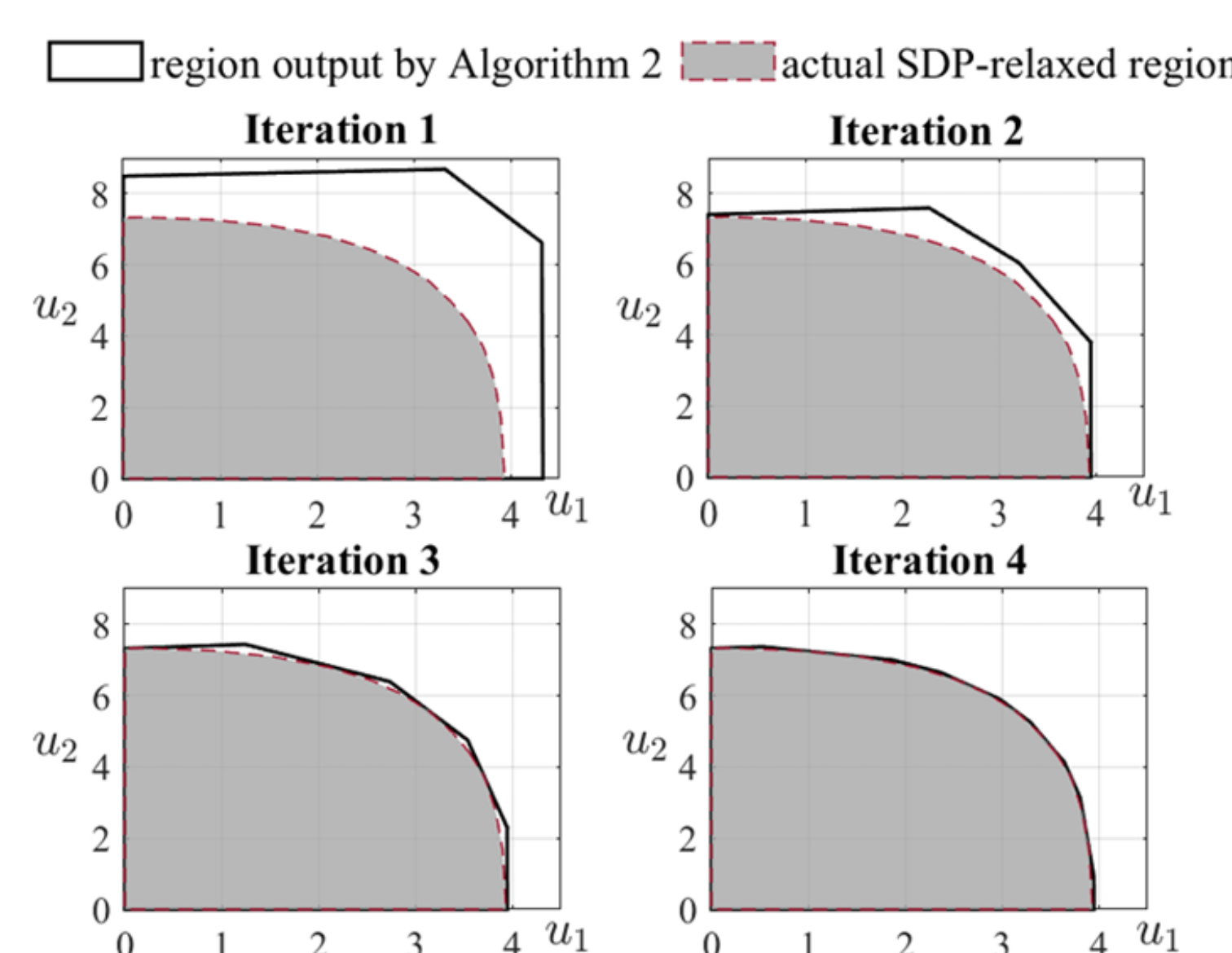
$$\begin{aligned}
 & \text{rank}(W) = 1 \\
 & \uparrow \text{KKT conditions} \\
 & \dim(\text{null}(B)) = 1
 \end{aligned}$$

β : the weighting factor of the sum of slack variables

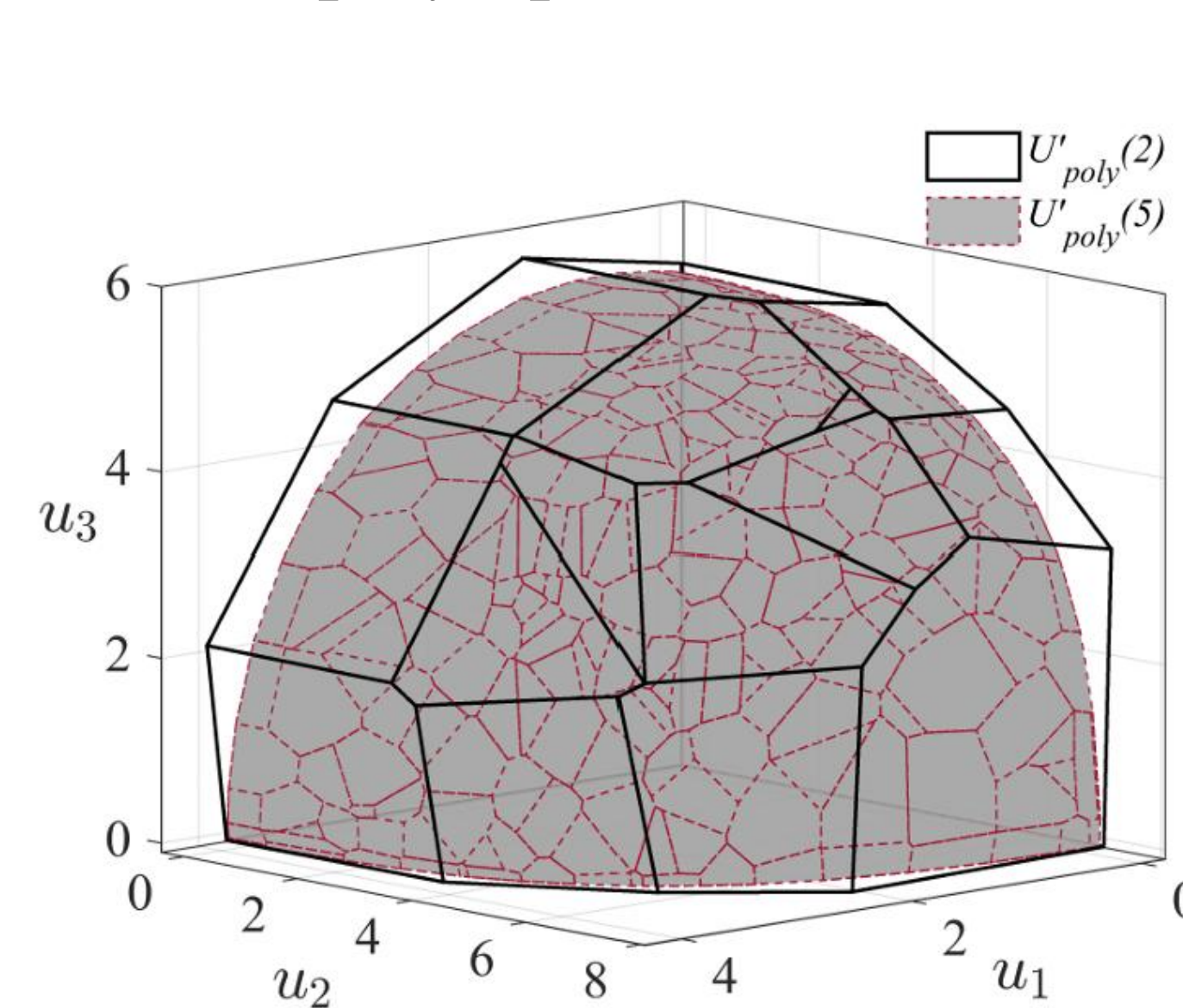


Numerical results

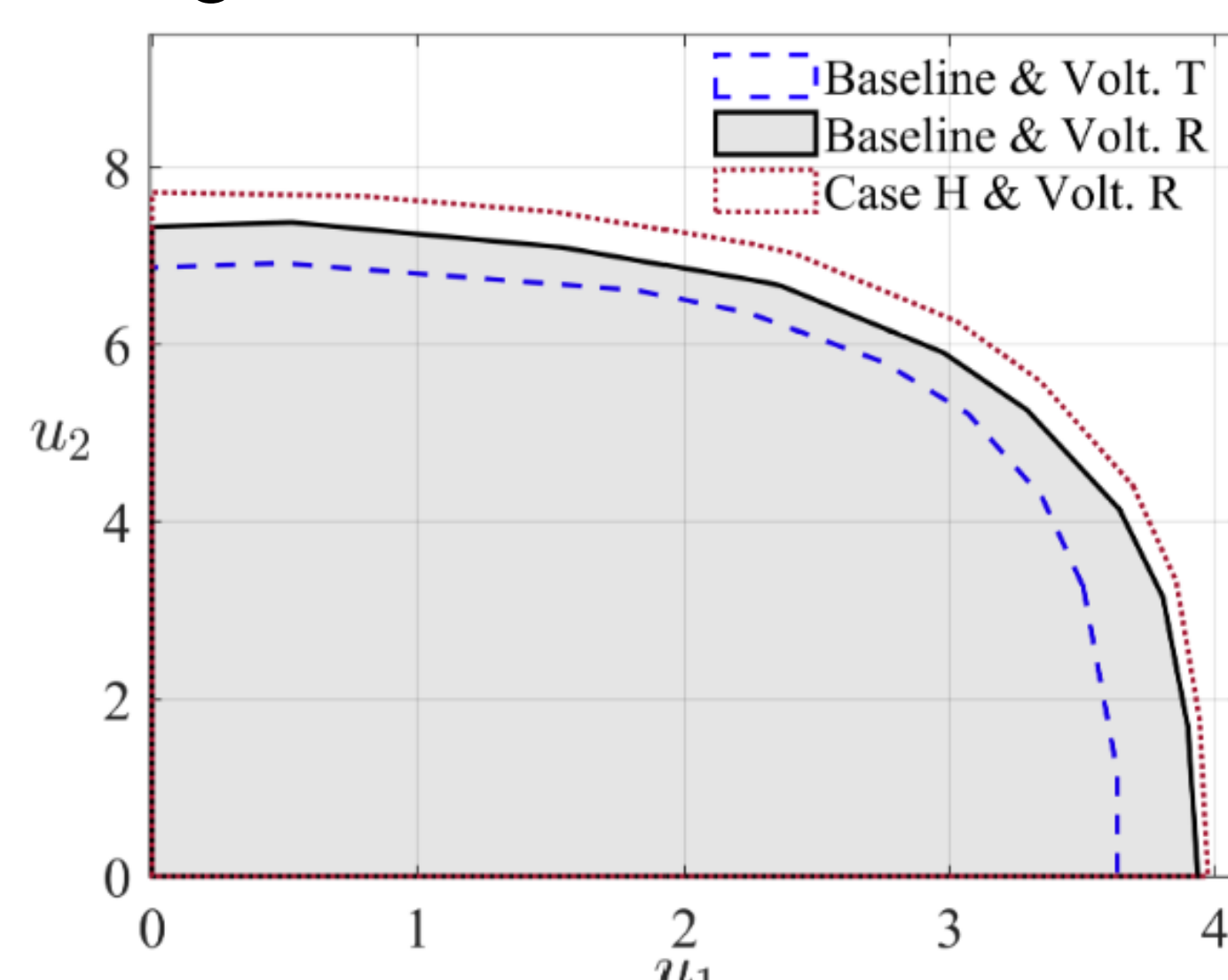
2D polytope across iterations



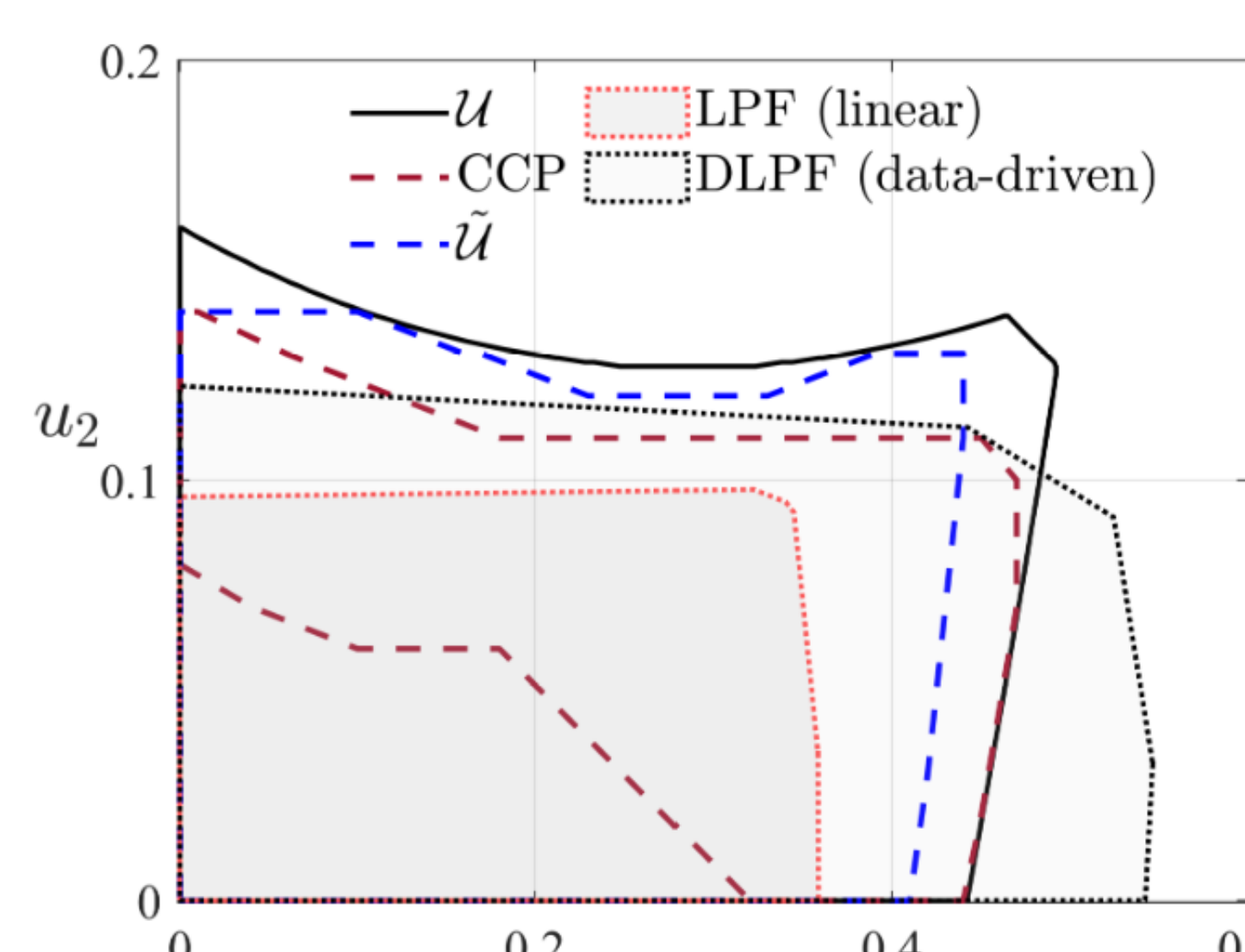
3D polytope across iterations



Impact of voltage and power injection ranges

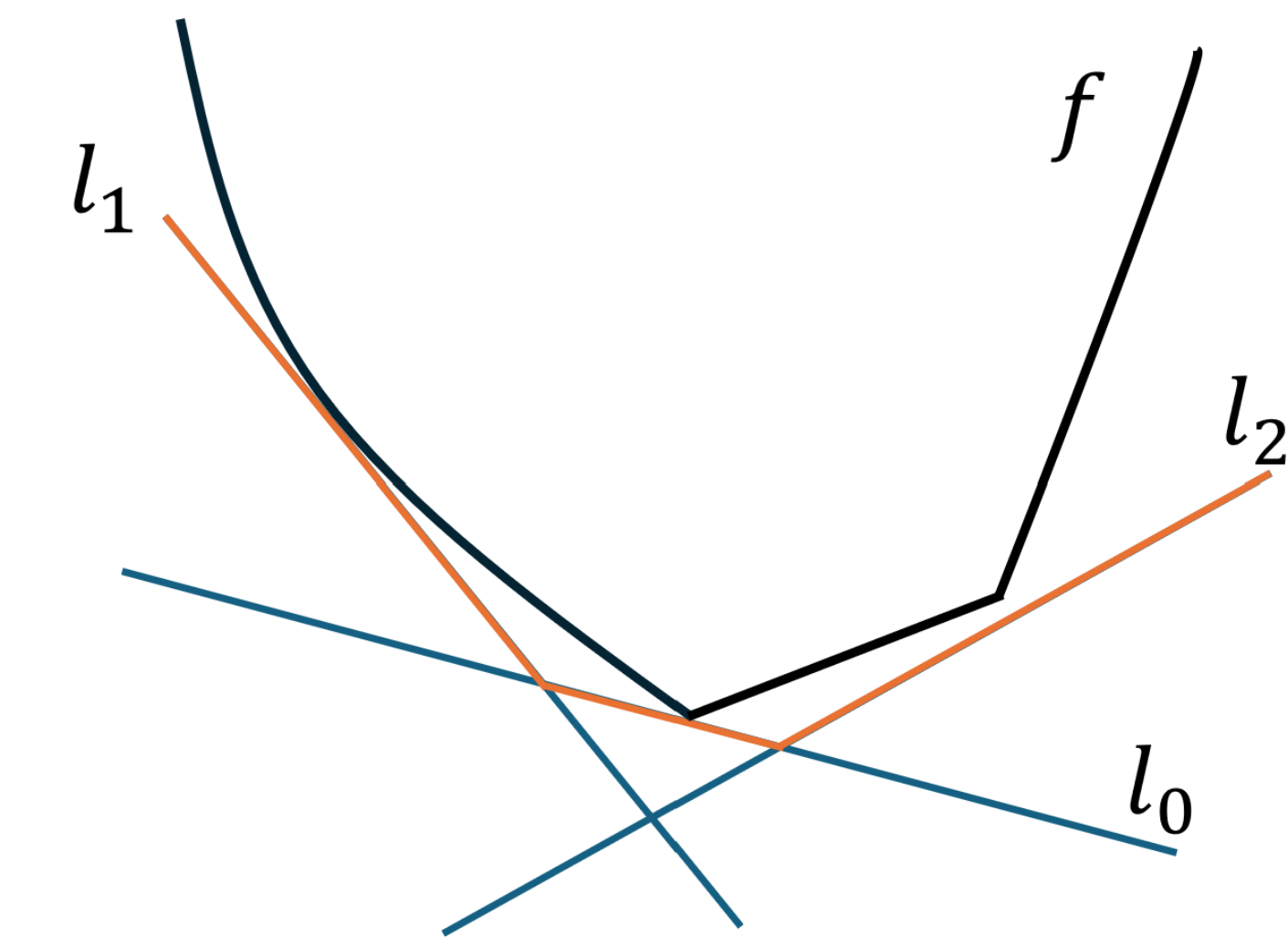


Different models comparison



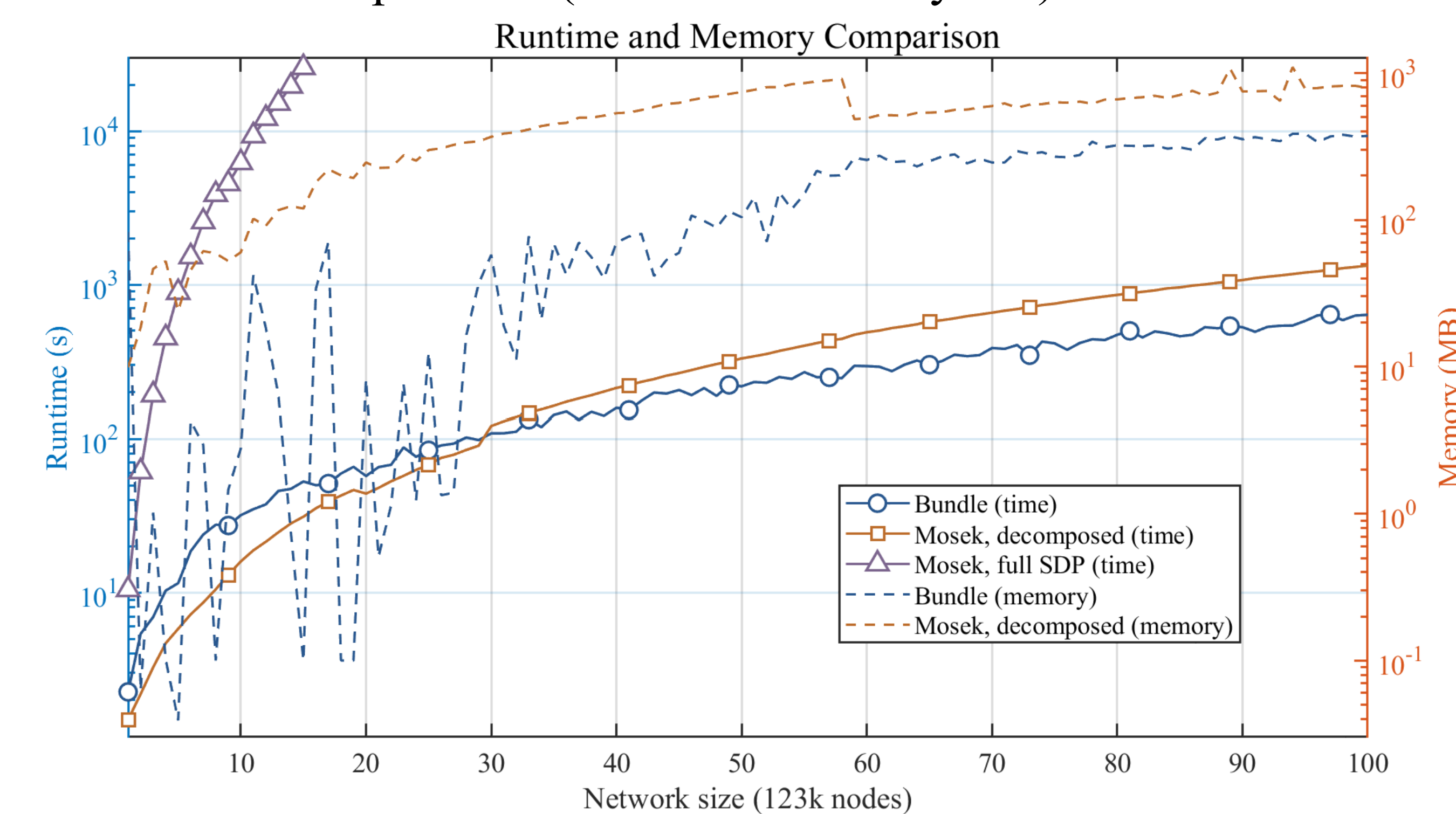
Bundle method

- Scalable solution approach:
 - Avoiding expensive primal dual interior-point iterations
 - Targeting on large-scale networks such as 123*k



The penalized objective function f , which penalizes violation of the PSD constraint, is non-smooth. We approximate it by three cutting planes, which together form a piece-wise linear lower model of f : a fixed plane, a current plane generated from a subgradient of f , and an aggregate plane.

- Performance comparisons (Time and memory use)



Conclusions

Dispatchable region computation for **unbalanced three-phase AC** network:

- Outer polytope approximation
- Sufficient condition for SDP exactness, leading to a pointwise inner approximation
- Bundle method solver for scalable OPF/SDP computation

Publications

- B. Fang, C. Zhao, and S. Low, "Convergence of Backward/Forward Sweep for Power Flow Solution in Radial Networks," 2023 62nd IEEE Conference on Decision and Control (CDC), Singapore.
- B. Fang, C. Zhao, and S. Low, "Convergence of Backward/Forward Sweep for Power Flow Solution in Radial Networks," in IEEE Transaction on Control of Network Systems, vol. 12, no. 2, pp. 1780-1792, 2025.
- B. Fang, C. Yue, and C. Zhao, "Approximating Dispatchable Regions in Three-Phase Radial Networks with Conditions for Exact SDP Relaxation," in IEEE Transactions on Power Systems (minor revision).
- B. Fang, L. Ding, and C. Chen, "A Bundle Method for Feasibility in Semidefinite Relaxations of Unbalanced Three-Phase OPF", Ongoing Research, 2025.

Open to Collaboration

Bohang Fang, Postdoctoral Researcher, Dartmouth College

Research interests: optimization in power system, unbalanced three-phase AC networks, convex relaxation, scalable OPF/SDP solvers, and AI/LLM for power system.

Interested in collaborations on:

- Power system optimization and control
- Large-scale OPF/SDP algorithms
- AI/LLM for energy system

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